

Dialectica Categories Revisited

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Abstract

Gödel's Dialectica interpretation was conceived as a tool to obtain the consistency of Peano arithmetic via a proof of consistency of Heyting arithmetic, in the 40s. In recent years, several proof-theoretic transformations, based on Gödel's Dialectica interpretation, have been used systematically (by Kohlenbach and many others) to extract new content from classical proofs, following a suggestion of Kreisel. Thus, the interpretation has found new relevant applications in several areas of mathematics and computer science.

Several authors have explained the Dialectica interpretation in categorical terms. In particular, I have introduced the notion of a Dialectica category as an internal version of Gödel's Dialectica Interpretation in my doctoral thesis, written under Hyland's supervision. This categorical Dialectica construction has been generalized in many meaningful directions, and it has had many applications developed from it, from concurrency theory and Petri nets, from linear logic models of state and games, to Set Theory and 'small cardinals' and 'Kolmogorov-Veloso problems'. Recently the construction has been under scrutiny, as many applications in computing, especially ones using lenses and bidirectional transformations have been discussed, in parallel with applications of the category of polynomials. This is all part of a growing movement of Applied Category Theory, of which the Topos Institute is one of the centers and that we shall also discuss.

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