# A foundation in which numbers count

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**Abstract**

The notion of number employed in everyday discourse is bound up with the act of counting. Despite this, neither of the two major foundational approaches to arithmetic give a full account of what is involved, conceptually, with the act. The Dedekind-Peano axioms capture the sequential character of the numbers we count with, but do not illuminate what we determine in counting with them. Conversely the how-many questions we answer with counting are given center stage in neo-Fregean approaches, but the central role of number sequences in answering them via counting goes unacknowledged. Taking remarks of Benacerraf in his seminal [1] as a starting point, I present a foundational approach to arithmetic that charts a course between these two poles. Numbers at the most basic level are conceptualized as words that appear in acts of what Benacerraf terms 'intransitive counting.' From this basis a notion of a counting sequence is developed, in terms of which the operations of addition and multiplication are defined, In the resulting theory, the generality of statements such as the commutativity of addition are secured not by mathematical induction, but a version of the pigeonhole principle.

## References

[1] Benacerraf, P.; What numbers could not be, *The Philosophical Review* 74:47–73, 1965.

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